

Simulating a PTA with metronomes and microphones:  
A user's guide for a double-metronome timing & correlation  
demonstration

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## I Purpose

The purpose of the double-metronome timing and correlation demonstration is to illustrate (using two metronomes and a microphone) how a pulsar timing array (PTA) uses correlation methods to search for gravitational-wave signals in pulsar timing data. As a gravitational wave transits the line-of-sight between the Earth and an array of pulsars in our galaxy, it induces correlations in the measured timing residuals across pairs of pulsars in the array. In this demonstration, radio pulses from an array of pulsars are represented by ticks of an array of metronomes (only two metronomes are needed for this demo); radio receivers on Earth are represented by a single microphone; and the passage of a gravitational wave is represented by the motion of the microphone (basically, the “Earth-term” in pulsar timing language). The motion of the microphone Doppler shifts the received frequency of the metronome pulses, changing the arrival times of these pulses at the microphone from that in the absence of microphone motion.

This demonstration is best suited for undergraduates or senior-level highschool students. It illustrates the following techniques used in pulsar timing analyses:

1. Estimate the pulse period and calculate a reference pulse profile (i.e., a template) by folding data consisting of several pulses.
2. Estimate the times of arrival (TOAs) of the pulses by correlating the template pulse profile with the data.
3. Calculate the timing residuals by subtracting the expected TOAs (assuming a simple timing model) from actual TOAs, which were estimated in the previous step.
4. Improve the estimate of the pulse period by removing a linear trend from the timing residuals.
5. Show that the timing residuals for a pair of pulsars are correlated as a function of the angular separation between the lines-of-sights to the two pulsars.

For less-mathematically-inclined audiences, the mathematical description of the demonstration will need to be reduced accordingly. But the visual representation of the residuals from the two metronomes being shifted in phase relative to one another by an amount equal to their angular separation should be comprehensible to all audiences.

## II Required hardware

Two metronomes are required, Seiko model SQ50-V quartz metronome being the preferred choice. This model has adjustable beats-per-minute (bpm) up to 208 bpm, adjustable volume, and two different tempo sounds (mode *a* and mode *b*, with mode *b* having a slightly higher pitch). Having two modes is helpful in distinguishing the pulses from the individual metronomes, since the pulse profiles are different.

One also needs some type of microphone, either an external USB microphone or an internal microphone, connected to a laptop that is set up to run the relevant data analysis routines (see Secs. III, IV, V for details). I have found that the internal microphone on my laptop works best since it has ambient noise reduction, although it is somewhat inconvenient (awkward) to physically move the laptop to simulate the passage of a gravitational wave. To produce uniform circular motion of the microphone, one can either swing an external USB microphone in a circle supporting it by its cable, or place the laptop on a rotating platform that can turn freely.

Michele Vallisneri suggested doing this demo using three cellphones: two acting as pulsars by running metronome apps, and the other acting as the radio receivers on Earth, behaving as a microphone and running a javascript implementation of the data analysis routines.

In addition, one needs an open space covering an area of about 10 ft × 5 ft for the placement of the two metronomes and microphone. As schematic diagram of the setup is shown in Figure 1. A photograph of an actual real-world setup used to take the data described in this user’s guide is shown in Figure 2.

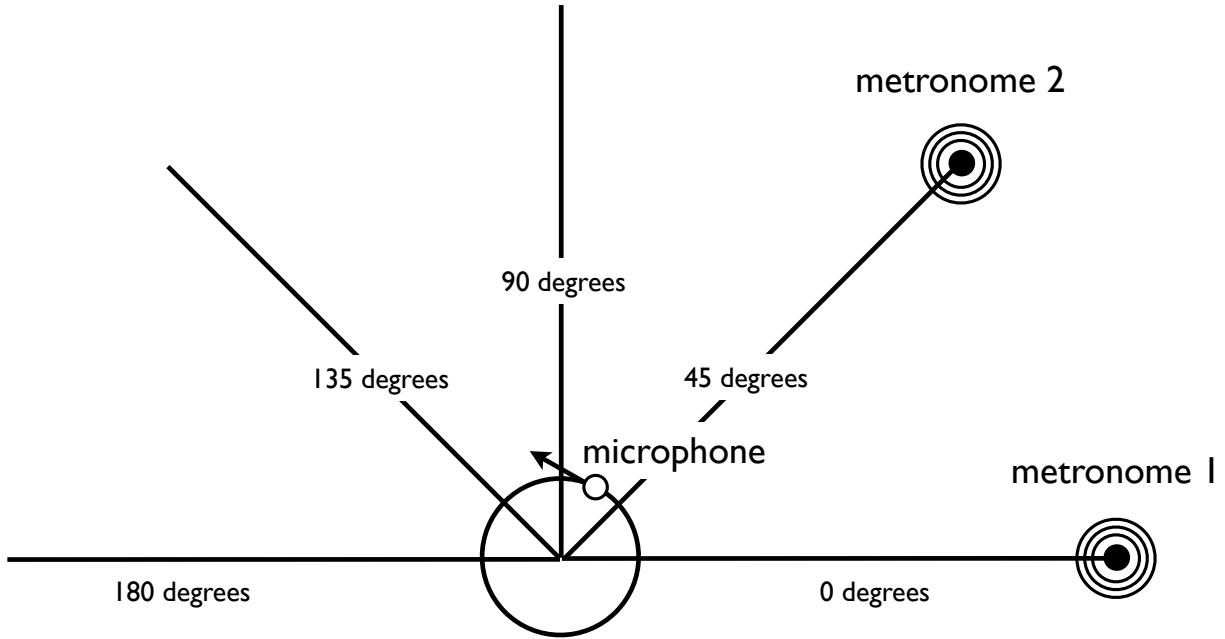


Figure 1: Schematic diagram showing the location of the microphone and metronomes for the different analyses. The stationary microphone is located at the origin; the moving microphone undergoes uniform circular motion, indicated by the counter-clockwise circle. Metronome 2 is placed at angular location  $45^\circ$  in this figure, but will be placed at the other angular locations for different parts of this demo.



Figure 2: Photograph of an actual setup for data taking. Metronome 1 is shown at angular location  $0^\circ$ ; metronome 2 at angular location  $135^\circ$ . The separation between the microphone (located at the origin) and the metronomes at the edge of a semi-circle is approximately 5 feet.

### III Required software

Both Matlab and Python-based routines exist for doing the data analysis. These includes: (i) audio recording and play back of metronome pulses, (ii) data folding, (iii) pulse profile calculation, (iv) matched-filtering estimation of pulse arrival times, (v) timing residual calculation, (vi) linear detrending of timing residuals, (vii) fitting of sinusoids to the timing residuals, and (viii) correlation coefficient calculation. Two Python-based GUIs exist for performing the two main data taking and data analysis tasks: `PTAdemo0GUI.py` (for analyzing the single-metronome pulse data) and `PTAdemo2GUI.py` (for analyzing the double-metronome pulse data). These two GUIs and some of the underlying routines are described in detail in the following sections.

### IV Single-metronome pulse analysis

#### A The GUI: `PTAdemo0GUI.py`

A screenshot of `PTAdemo0GUI.py` is shown in Figure 3. The GUI has space for plots of: (i) pulses from the individual metronomes, (ii) pulse profiles (obtained by folding the pulse data), and (iii) timing residuals. The GUI also has several text entry fields and buttons, whose functions are described below:

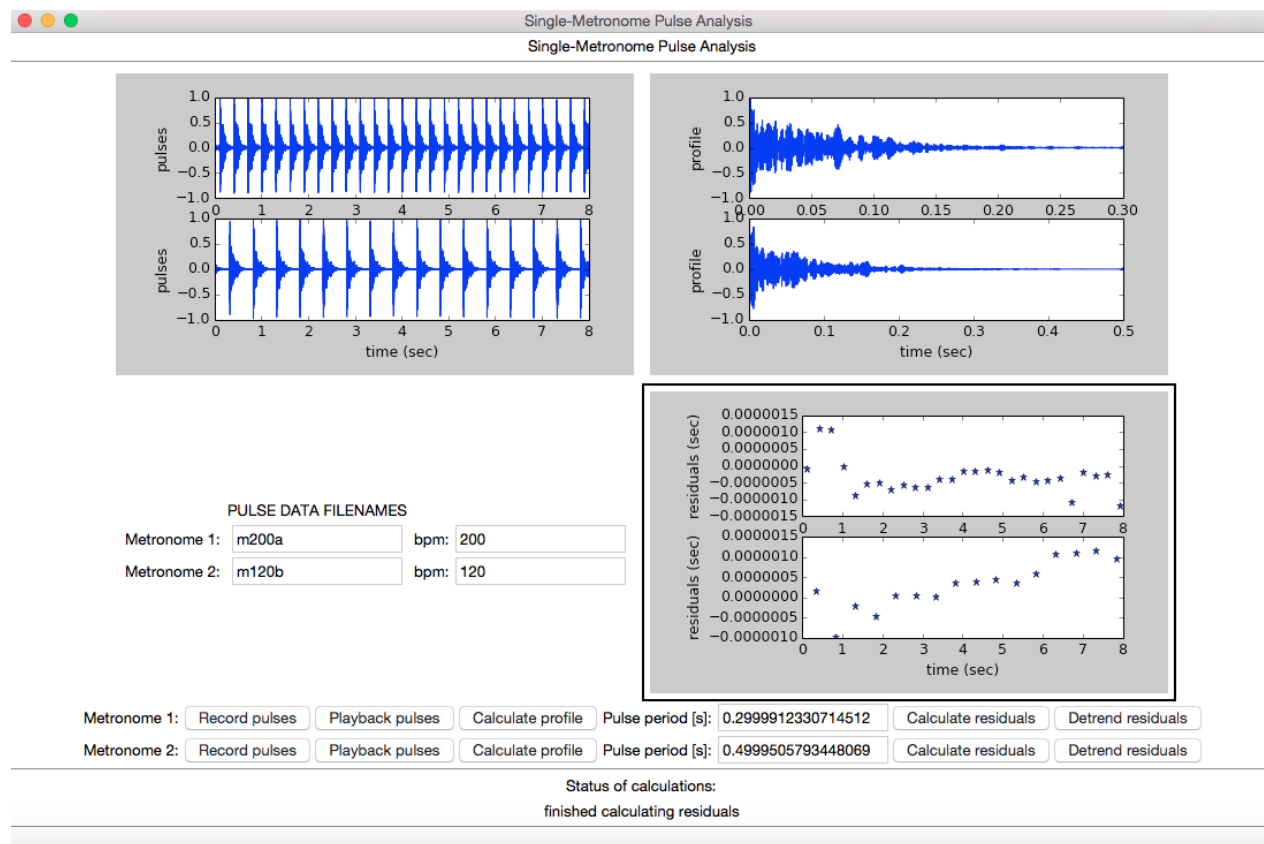


Figure 3: Python GUI for the single-metronome pulse analysis.

**Record pulses:** Records audio pulse data from a metronome, and save the corresponding timeseries to an ascii `.txt` file with file prefix specified by the 'PULSE DATA FILENAMES' text entry boxes (default `m200a` or `m120b`). The 'bpm' text entry boxes have the beats-per-minute settings for the two metronomes. The pulse recording routine is hard-coded to record 6 seconds of data.

**Playback pulses:** Plays back and plots the audio pulse data saved in `m200a.txt` or `m120b.txt`.

**Calculate profile:** Either (i) calculates the pulse period  $T_p$  and pulse profile directly by folding the metronome data, and then saves the profile to the file `m200a_profile.txt` or `m120b_profile.txt`, or (ii) reads in the pulse profile data that has already been saved in these files. Method (i) is used only the first time the analysis is run. If the pulse profiles are read-in from the files `m200a_profile.txt` and `m120b_profile.txt`, the text entry boxes for the pulse periods need to be entered by hand. For both cases, the pulse profile is plotted from 0 to  $T_p$ .

**Calculate residuals:** Calculates the timing residuals by subtracting the expected TOAs from the measured TOAs of the pulses:

$$\text{residual}(i) = \text{measured}(i) - \text{expected}(i) \quad (1)$$

The expected arrival times are defined by a simple timing model

$$\text{expected}(i) = \text{measured}(i_0) + (i - i_0)T_p \quad (2)$$

which is just the measured TOA of the pulse having the largest correlation (see below) plus integer multiples of the pulse period  $T_p$  of the metronome. The measured TOAs are obtained by correlating time-shifted pulse profiles with the data. Mathematically, the correlation function is defined by

$$C_I(t) = \mathcal{N}_I \int_0^{f_{\text{Nyq}}} df e^{i2\pi ft} \tilde{y}(f) \tilde{p}_I^*(f), \quad \mathcal{N}_I = \left[ \int_0^{f_{\text{Nyq}}} df |\tilde{p}_I^*(f)|^2 \right]^{-1} \quad (3)$$

where  $\tilde{y}(f)$  is the Fourier transform of the measured pulse data  $y(t)$ , and  $\tilde{p}_I(f)$  is the Fourier transform of the template pulse profiles  $p_I(t)$  for the two metronomes,  $I = 1, 2$ .  $C_I(t)$  will have local maxima at the arrival times of the pulses. The normalization constant  $\mathcal{N}_I$  is included so that the values of the correlation function at the estimated arrival times are estimates of the amplitudes of the pulses.

**Detrend residuals:** Improves the estimate of the pulse period for a metronome by removing a linear trend from the timing residuals, if such a trend exists. Detrending may change the estimate of the pulse period by 1-2 microseconds. The updated period is displayed in the ‘Pulse period’ text entry box.

## B Steps for doing the analysis

1. Record pulses from each metronome separately, keeping the microphone stationary. The microphone should be located at the origin of coordinates and the two metronomes should be at angular location  $0^\circ$ . The file prefixes and bpm/s in the text entry boxes should be chosen to match the physical settings of the metronome.
2. After recording the pulse data for each metronome, you can play it back, calculate the corresponding pulse profile and period, calculate the residuals, and detrend the residuals, by simply pressing the relevant GUI buttons.

This analysis produces two pulse profile data files (e.g., `m200a_profile.txt` and `m120b_profile.txt`) and the associated pulse periods  $T_1, T_2$ , which are needed for the double-metronome pulse analysis described in the next section.

## V Double-metronome pulse analysis

### A The GUI: `PTAdemo2GUI.py`

A screenshot of `PTAdemo2GUI.py` is shown in Figure 4. The GUI has space for plots of: (i) pulses from the two metronomes running simultaneously, (ii) reference pulse profiles for the two metronomes, which were calculated using `PTAdemo0GUI.py`, and (iii) timing residuals for the two metronomes. The GUI also has several text entry fields and buttons, whose functions are described below:

**Record pulses:** Records audio pulse data from the two metronomes running simultaneously, and then saves the corresponding timeseries to an `ascii.txt` file with file prefix specified by the ‘Data file’ text entry box under the ‘FILENAMES’ label (default `m200a120b0`).

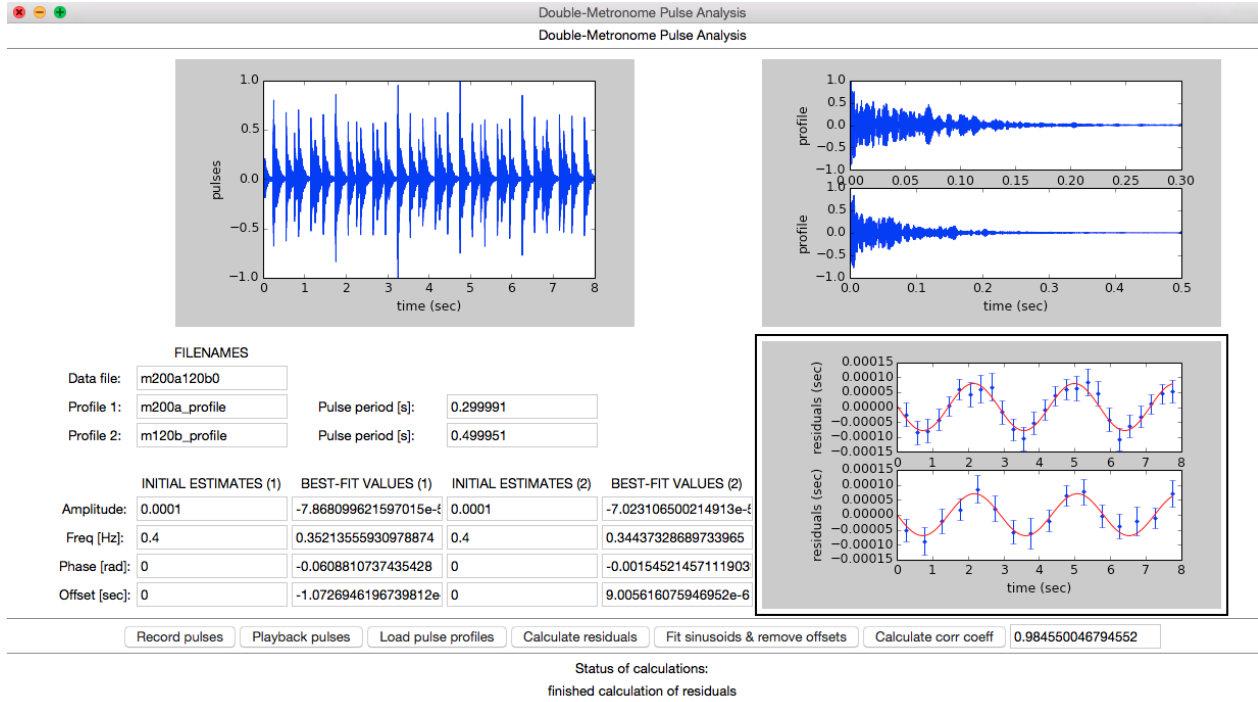


Figure 4: Python GUI for the double-metronome pulse analysis.

**Playback pulses:** Plays back and plots the audio pulse data saved in `m200a120b0.txt`.

**Load pulse profiles:** Reads-in and plots the reference pulse profiles for the two metronomes, which were calculated by `PTAdemo0GUI.py` and were saved in `ascii.txt` files specified by the 'Profile 1,2' text entry boxes under the 'FILENAMES' label (default `m200a_profile.txt` and `m120b_profile`). The text entry boxes for the pulse periods should be filled with in with the values calculated by `PTAdemo0GUI.py`.

**Calculate residuals:** Calculates the timing residuals as described previously for `PTAdemo0GUI.py`.

**Fit sinusoids & remove offsets:** Simultaneously removes constant offsets and calculates best-fit sinusoids to the timing residuals for the two metronomes, using initial estimates for the amplitude, frequency, and phase of the sinusoids, and the constant offset given in the text entry boxes labeled 'INITIAL ESTIMATES (1,2)' (defaults  $1 \times 10^{-4}$ , 0.5 Hz, 0 radians, and 0 sec). The constant offset arises from the arbitrariness of setting the timing residual of the pulse with the highest correlation to zero. The best-fit parameter values calculated by 'Fit sinusoids & remove offsets' are written to the text entry boxes labeled 'BEST-FIT VALUES (1,2)'.

**Calculate corr coeff:** Calculates the time-averaged correlation coefficient between the best-fit sinusoids for the two sets of timing residuals. If we denote the best-fit sinusoids by  $x(t)$  and  $y(t)$ , then

$$\text{corr coeff} \equiv \frac{\langle xy \rangle}{\sqrt{\langle x^2 \rangle \langle y^2 \rangle}}, \quad \text{where} \quad \langle xy \rangle \equiv \frac{1}{T} \int_0^T dt x(t)y(t) \quad (4)$$

Theoretically, the value of the correlation coefficient should equal  $\cos \chi$ , where  $\chi$  is the separation angle between the line-of-sights to the two metronomes as described in Section VI.

## B Steps for doing the analysis

1. Start by placing both metronomes at the same angular location  $0^\circ$  and at the same distance  $L \sim 1$  m from the origin. With both metronomes running simultaneously, record the audio data while setting

the microphone in uniform circular motion about the origin:

$$\vec{r}(t) = A [\cos(2\pi f_0 t + \phi_0) \hat{x} + \sin(2\pi f_0 t + \phi_0) \hat{y}] \quad (5)$$

Typically, it is best to have  $A \approx 10$  cm ( $=0.1$  m) and period of oscillation  $T_0 \equiv 1/f_0 \approx 2$  s. This leads to microphone-induced timing residuals of order  $A/c \approx 3 \times 10^{-4}$  s, where  $c = 340$  m/s is the speed of sound in air. (See Section VI for a detailed discussion of microphone-induced timing residuals.) This turns out to be more than an order-of-magnitude larger than the precision to which we can estimate the TOAs of the metronome pulses, meaning that we can easily observe the effect of microphone motion in the timing-residual data.

2. After recording the double-metronome data, you can play it back, load the pulse profiles, and calculate and detrend the residuals for each metronome. The timing residuals induced by the microphone motion should be sinusoidal and have large signal-to-noise ratio. You should then fit sinusoids to the residuals for each metronome, adjusting the 'INITIAL GUESS' amplitudes, frequencies, and phases if necessary. (The initial guesses just have to be close, not exact.) Finally, you should calculate the correlation coefficient, which should have a value very close to 1 for this case, since the two microphones are at the same angular location.
3. Repeat the above two steps but with metronome 2 at different angular locations ( $45^\circ$ ,  $90^\circ$ ,  $135^\circ$ , and  $180^\circ$ ) with respect to metronome 1 (which should always remain at  $0^\circ$ ). The motion of the microphone should be as similar as possible to that for Step 1. Change the name of the file prefix in the 'Data file' text entry box to `m200a120bXX`, where `XX` is 45, 90, 135, and 180, to reflect the change in the angular location of metronome 2. You should find that the correlation coefficient is approximately equal to  $\cos \chi$ , where  $\chi = 45^\circ, 90^\circ, 135^\circ$  and  $180^\circ$  is the angular separation of the two metronomes.

This analysis produces data files (`m200a120bXX.txt`, where `XX` is 0, 45, 90, 135, and 180), containing the double-metronome pulse timeseries.

## VI Discussion

### A Microphone-induced timing residuals

Theoretically, the timing residuals for metronome  $I = 1, 2$  induced by a moving microphone are given by

$$\text{residual}_I(t) \equiv \frac{\Delta L_I(t)}{c} = \frac{L_I(t) - L_I}{c} \simeq -\frac{1}{c} \hat{u}_I \cdot \vec{r}(t) \quad (6)$$

where  $L_I(t)$  is the distance between metronome  $I$  and the location of the microphone  $\vec{r}(t)$  at time  $t$ , and  $L_I$  is the nominal distance between the metronome and the microphone. This is just the change in the metronome pulse-propagation time due to the motion of the microphone relative to metronome  $I$ . The last (approximate) equality above is valid if we ignore correction terms of order  $A/L_I \sim 0.1$ , where  $A \approx 5$  cm is the amplitude of the microphone motion and  $L_I \approx 1$  m is the distance from the metronome to the microphone when it is at the origin. These correction terms arise from the finite distance of the metronome from the microphone; in other words, the pulse wavefronts are actually curved not planar. Including the next-order correction term gives us

$$\text{residual}_I(t) \simeq -\frac{1}{c} \left[ \hat{u}_I \cdot \vec{r}(t) + \frac{1}{2L_I} ((\hat{u}_I \cdot \vec{r}(t))^2 - |\vec{r}(t)|^2) \right] \quad (7)$$

For the microphone undergoing uniform circular motion (5):

$$\text{residual}_I(t) \simeq -\frac{A}{c} \cos(2\pi f_0 t + \phi_0 - \zeta_I) \quad (8)$$

where  $\zeta_I$  is the angle that the location of metronome makes with respect to the  $x$ -axis, and where we have ignored the higher-order correction terms to the residual. Using the trig identity

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)] \quad (9)$$



it is fairly easy to show that the time-averaged correlation coefficient of the microphone-induced timing residuals for metronomes 1 and 2 is

$$\text{corr coeff} \simeq \cos \chi \tag{10}$$

where  $\chi \equiv \zeta_1 - \zeta_2$  is the angle between the line-of-sights to the two metronomes. This equality is correct to order  $(A/L)^2$ . (Since the timing residuals for the two metronomes are evaluated at different times, we actually correlate the best-fit sinusoids to the timing residuals.) This dependence of the correlation coefficient on the angle between the two metronomes is confirmed (roughly) by the double-metronome analysis.

## **B Technical remarks about uniform circular motion**

We have chosen uniform circular motion for the microphone and not (linear) sinusoidal motion in a plane, since the angular dependence of the correlation coefficient for linear motion would depend not only on the angular separation  $\chi$  of the two metronomes, but also on their angular separation with respect to the plane of oscillation of the microphone. In addition, any linear sinusoidal motion can be written as a superposition of counter-clockwise and clockwise uniform circular motions. In fact, *any* motion whatsoever can be written as a linear superposition of sinusoidal motions having different frequencies and planes of oscillations, as a consequence of Fourier's theorem. Thus, there is no loss in generality in considering the simple uniform circular motion case.

Finally, we note that the effect of uniform circular motion on the timing residuals is exactly what one sees in real pulsar-timing timing residuals if the yearly orbital motion of the Earth around the Sun is not taken into account in the timing model for the pulsars.